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Hertz–Debye potentials and pulse propagation

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Abstract. Hertz–Debye potentials are generally used for time-harmonic fields. We are interested here in arbitrary time-dependent pulses tackled with the help of the Laplace transform, and in addition, we assume that propagation takes place in a chiral reciprocal medium. We show how the Hertz–Debye potentials work to solve the Laplace transformed Maxwell equations and illustrate this formalism with three applications. Finally, we discuss how to obtain the solution in the time domain through the inverse Laplace transform.

1. Introduction

Hertz potentials are a well known tool [1] for solving Maxwell's equations. We discuss here how these potentials can be used to analyse the structure of electromagnetic pulses, originating at some time t_0, depending arbitrarily on time and propagating in a dispersive isotropic reciprocal medium. The old Sommerfeld–Brillouin problem [2, 3] is the prototype of the pulses considered in this work. In mathematical terms, one has to deal with initial value problems and as stated by Stratton [1], the Laplace transform

$$F(s) = \int_0^\infty \mathrm{e}^{-st} f(t)\,\mathrm{d}t \qquad \mathrm{Re}\, s > 0 \tag{1}$$

was introduced for the purpose of treating this kind of problem.

These pulses propagate in a medium with the constitutive relations

$$\begin{aligned} \boldsymbol{D}(\boldsymbol{r}, t) &= \int_0^\infty \varepsilon(\tau)\boldsymbol{E}(\boldsymbol{r}, t-\tau)\,\mathrm{d}\tau + \int_0^\infty \xi(\tau)\boldsymbol{B}(\boldsymbol{r}, t-\tau)\,\mathrm{d}\tau \\ \boldsymbol{H}(\boldsymbol{r}, t) &= \int_0^\infty \mu^{-1}(\tau)\boldsymbol{B}(\boldsymbol{r}, t-\tau)\,\mathrm{d}\tau - \int_0^\infty \xi(\tau)\boldsymbol{E}(\boldsymbol{r}, t-\tau)\,\mathrm{d}\tau \end{aligned} \tag{2}$$

where $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{D}$ and $\boldsymbol{H}$ are the usual components of the electromagnetic field and ε, μ and ξ are the permittivity, permeability and chirality of the medium, respectively. Applying (1) to (2) gives (we use the same symbol for a function and its transform)

$$\begin{aligned} \boldsymbol{D}(r, s) &= \varepsilon(s)\boldsymbol{E}(r, s) + \xi(s)\boldsymbol{B}(r, s) \\ \boldsymbol{B}(r, s) &= \mu(s)\boldsymbol{H}(r, s) + \mu(s)\xi(s)\boldsymbol{E}(r, s). \end{aligned} \tag{3}$$

In the Sommerfeld–Brillouin problem $\xi = 0$, $\mu = 1$, $\varepsilon(s) = 1 + a^2(s^2 + bs + s_0^2)^{-1}$ [1].

With the electromagnetic field null for $t \leqslant 0$, the Laplace transformed Maxwell equations are

$$\begin{aligned} &\mathrm{curl}\,\boldsymbol{E}(\boldsymbol{r}, s) = -\frac{s}{c}\boldsymbol{B}(\boldsymbol{r}, s) \qquad \mathrm{curl}\,\boldsymbol{H}(\boldsymbol{r}, s) = \frac{s}{c}\boldsymbol{D}(\boldsymbol{r}, s) \\ &\mathrm{div}\,\boldsymbol{D}(\boldsymbol{r}, s) = \mathrm{div}\,\boldsymbol{B}(\boldsymbol{r}, s) = 0. \end{aligned} \tag{4}$$

Substituting (3) into (4) gives the system of equations to be solved in the s-domain and we show how the Hertz potential technique works to obtain the solution. However, we first present a complex formalism making the discussion of Maxwell's equations easier.

2. Complex field vector

Let us consider the two complex vectors [4] ($\mathrm{i} = \sqrt{-1}$)

$$\boldsymbol{P}(\boldsymbol{r}, s) = a(s)\boldsymbol{E}(\boldsymbol{r}, s) + \mathrm{i}b(s)\boldsymbol{H}(\boldsymbol{r}, s) \qquad \boldsymbol{Q}(\boldsymbol{r}, s) = b(s)\boldsymbol{D}(\boldsymbol{r}, s) + \mathrm{i}a(s)\boldsymbol{B}(\boldsymbol{r}, s) \tag{5}$$

in which $a(s)$ and $b(s)$ are two functions to be determined. With (5), equations (4) become

$$\operatorname{curl} \boldsymbol{P}(\boldsymbol{r}, s) = \mathrm{i}\frac{s}{c}\boldsymbol{Q}(\boldsymbol{r}, s) \qquad \operatorname{div} \boldsymbol{P}(\boldsymbol{r}, s) = 0. \tag{6}$$

We now prove that in a medium with the constitutive relations (3), we may define $a(s)$, $b(s)$ and $m(s)$ so that equation (6) can be written

$$\operatorname{curl} \boldsymbol{P}(\boldsymbol{r}, s) = \mathrm{i}sm(s)/c\boldsymbol{P}(\boldsymbol{r}, s) \qquad \operatorname{div} \boldsymbol{P}(\boldsymbol{r}, s) = 0. \tag{7}$$

From now on for simplification, we no longer write the arguments $\boldsymbol{r}$, s. Substituting (3) into the definition of $\boldsymbol{Q}$ (5) gives us with $d = \varepsilon + \mu\xi^2$

$$\boldsymbol{Q} = (bd + \mathrm{i}a\mu\xi)\boldsymbol{E} + (b\mu\xi + \mathrm{i}a\mu)\boldsymbol{H} \tag{8}$$

and the condition $\boldsymbol{Q} = m\boldsymbol{P}$ produces the homogeneous system of equations

$$(m - \mathrm{i}\mu\xi)a - bd = 0 \qquad -\mu a + (m + \mathrm{i}\mu\xi)b = 0 \tag{9}$$

with non-trivial solutions if $m^2 + \mu^2\xi^2 - \mu d = 0$ which implies $m = \pm(\varepsilon\mu)^{1/2}$. A solution of (9) is

$$a_\pm = \sqrt{\varepsilon} \pm \mathrm{i}\xi\sqrt{\mu} \qquad b = \pm\sqrt{\mu} \qquad m = \pm n \qquad n = \sqrt{\varepsilon\mu}. \tag{10}$$

Substituting (10) into the definition of $\boldsymbol{P}$ (5) gives

$$\boldsymbol{P}_\pm = (\sqrt{\varepsilon} \pm \mathrm{i}\xi\sqrt{\mu})\boldsymbol{E} \pm \mathrm{i}\sqrt{\mu}\boldsymbol{H} \tag{11}$$

and the Maxwell equations (7) take the simple form

$$\operatorname{curl} \boldsymbol{P}_\pm = \pm\mathrm{i}sn/c\boldsymbol{P}_\pm \qquad \operatorname{div} \boldsymbol{P}_\pm = 0. \tag{12}$$

Applying the terminology used in chemistry to classify chiral molecules, we could name enantiomers the two components $\boldsymbol{P}_+$, $\boldsymbol{P}_-$, of the electromagnetic field. However, in this complex formalism $\boldsymbol{P}_+$ and $\boldsymbol{P}_-$ are complex conjugate, so the mixture $\boldsymbol{P}_+\boldsymbol{P}_-$ is racemic, that is, the two enantiomers are present in equal amounts: we may therefore discard the subscript $\pm$ on complex quantities.

3. Complex potentials

3.1. Hertz potential

We look for solutions of equations (12) in terms of a vector potential $\boldsymbol{U}$ and we get

$$\boldsymbol{P} = \mathrm{i}\operatorname{curl}\boldsymbol{U} \qquad \mathrm{i}\operatorname{curl}\boldsymbol{U} = -ns/c\boldsymbol{U} + \operatorname{grad} V \tag{13}$$

in which V is a scalar gauge field. Then, if Π is a solution of the vector wave equation (Δ is the Laplacian operator) $\Delta\Pi - n^2s^2c^{-2}\Pi = 0$, as shown in the appendix, a simple calculation gives

$$\begin{aligned} \boldsymbol{U} &= -nsc^{-1}\boldsymbol{\Pi} + \mathrm{i}\operatorname{curl}\boldsymbol{\Pi} + \operatorname{grad} L \\ V &= -\operatorname{div}\boldsymbol{\Pi} + nsc^{-1}L. \end{aligned} \tag{14}$$

The complex vector $\mathbf{\Pi}$ is the Hertz potential and L is a scalar gauge that does not intervene in the expressions of the electromagnetic field. Substituting (14) into the first relation (13) gives the Righi–Whittaker formula [5]

$$\boldsymbol{P} = -\mathrm{curl}(\mathrm{curl}\,\mathbf{\Pi} + \mathrm{i}p\mathbf{\Pi}) \qquad p = ns/c \tag{15}$$

and using (11) (since $\boldsymbol{E}$ and $\boldsymbol{H}$ are real one has to restore the subscript $\pm$, $\Pi = \Pi_1 + \mathrm{i}\Pi_2$)

$$\begin{aligned} \sqrt{\varepsilon}\boldsymbol{E}_\pm &= -\mathrm{curl}(\mathrm{curl}\,\Pi_{1-} + p\Pi_2) \\ \sqrt{\mu}\mathsf{H}_\pm &= -\mathrm{curl}(\mathrm{curl}\,\Pi_2 \pm p\Pi_1) \qquad \mathsf{H} = \boldsymbol{H} + \xi\boldsymbol{E}. \end{aligned} \tag{16}$$

3.2. Hertz–Debye potentials

The previous expressions simplify considerably when the Hertz vector is radial, a frequent situation in practice. In this case, Π may be expressed in terms of a scalar Debye solution of the scalar wave equation $\Delta\psi - p^2\psi = 0$. Let (r, θ, ϕ) be the spherical coordinates and $\boldsymbol{a}_r, \boldsymbol{a}_\theta, \boldsymbol{a}_\phi$ the corresponding unit vectors, we then have

$$\begin{aligned} &\mathrm{curl}(r\boldsymbol{a}_r\psi) = [\partial_\phi(r\psi)/r\sin\theta]\boldsymbol{a}_\theta - [\partial_\theta(r\psi)/r]\boldsymbol{a}_\phi \\ &\mathrm{curl}[\mathrm{curl}(r\boldsymbol{a}_r\phi)] = [\partial_r^2(r\psi) - p^2r\psi]\boldsymbol{a}_r + [\partial_r\partial_\theta(r\psi)/r]\boldsymbol{a}_\theta + [\partial_r\partial_\phi(r\psi)/r\sin\theta]\boldsymbol{a}_\phi \\ &\qquad = -p^2r\boldsymbol{a}_r\psi + \mathrm{grad}\,L \qquad L = \partial_r(r\partial_r\psi). \end{aligned} \tag{17}$$

Substituting (17) into (15) yields a solution of Maxwell's equations (12) in terms of the scalar Debye potential $\Pi = r\boldsymbol{a}_r\psi$. We get from (16) with $\psi = \psi_1 + \mathrm{i}\psi_2$

$$\begin{aligned} \sqrt{\varepsilon}E_{r,\pm} &= -[\partial_r^2(r\psi_1) \mp rp^2\psi_1] \\ r\sqrt{\varepsilon}E_{\theta,\pm} &= -[\partial_r\partial_\theta(r\psi_1) \pm p\partial_\phi(r\psi_2)/\sin\theta] \\ r\sqrt{\varepsilon}E_{\phi,\pm} &= -[\partial_r\partial_\phi(r\psi_1)/\sin\theta \mp p\partial_\theta(r\psi_2)]. \end{aligned} \tag{18}$$

The components of $\sqrt{\mu}\mathsf{H}$ are obtained from (18) by changing $\psi_1, \pm\psi_2$ into $\psi_2, \mp\psi_1$.

Remark. The Hertz potentials are a powerful tool to find the electromagnetic field in a region throughout which ε and μ are constant, ρ and $\boldsymbol{J}$ equal to zero. However, to determine their physical significance, one has to relate them to their sources. Let P and M denote the electric and magnetic polarization vectors and let $\mathsf{Q} = \mathsf{P} + \mathrm{i}\mathsf{M}$, then the Hertz potential satisfies the inhomogeneous wave equation $\Delta\mathbf{\Pi} - p^2\mathbf{\Pi} = -\mathsf{Q}$ [1, 9].

4. Applications

Hertz and Hertz–Debye potentials have been used in the past mainly in the frequency domain [1, 5–12] for time harmonic fields and except for [11, 12] in non-chiral media. As states in the introduction, we are concerned here with initial value problems and arbitrary time-dependent pulses. We next give three applications of the formalism developed in section 3.

4.1. Radiation from an electric dipole

As an illustration of the previous remark, we consider a linear electric dipole $q(t)U(t)$ located at the origin of coordinates and vibrating in a fixed direction specified by the unit vector $\boldsymbol{a}_k$, $U(t)$ is the unit step function implying that the radiation is zero for $t \leqslant 0$. In a medium where the refractive index n and the chirality parameter ξ are constant, the Hertz vector is [9]

$$\Pi = r^{-1} q(s)\, \mathrm{e}^{-pr} \boldsymbol{a}_k \qquad p = ns/c \tag{19}$$

where r is the distance to the origin. Substituting (19) into (16) gives, with $q = q_1 + \mathrm{i} q_2$

$$\sqrt{\varepsilon}\boldsymbol{E}_\pm = A_1 q_1(s)(\boldsymbol{a}_k, \boldsymbol{a}_r)\boldsymbol{a}_r - A_2 q_1(s)\boldsymbol{a}_k \pm A_3 q_2(s)(\boldsymbol{a}_k \wedge \boldsymbol{a}_r). \tag{20}$$

Exchanging $q_1, \pm q_2$ into $q_2, \mp q_1$ in this expression gives $\sqrt{\mu}\mathbf{H}_\pm$ and

$$\begin{aligned} rA_1 &= (3r^{-2} - 3pr^{-1} + p^2)\, \mathrm{e}^{-pr} \\ rA_2 &= (r^{-2} + pr^{-1} + p^2)\, \mathrm{e}^{-pr} \\ rA_3 &= (pr^{-1} + p^2)\, \mathrm{e}^{-pr}. \end{aligned} \tag{20a}$$

Taking the k-direction as the z-axis, these relations reduce to ($\sqrt{\mu}\mathbf{H}$ is obtained as stated)

$$\begin{aligned} r^2\sqrt{\varepsilon}E_{r,\pm} &= 2(p + r^{-1})q_1(s)\, \mathrm{e}^{-pr} \cos\theta \\ r\sqrt{\varepsilon}E_{\theta,\pm} &= (r^{-2} + pr^{-1} + p^2)q_1(s)\, \mathrm{e}^{-pr} \sin\theta \\ r\sqrt{\varepsilon}E_{\phi,\pm} &= \pm p(p + r^{-1})q_2(s)\, \mathrm{e}^{-pr} \sin\theta. \end{aligned} \tag{21}$$

One observes at once that when $q(s)$ is real the only non-null components are E_r, E_θ, H_ϕ, and that the radial field does not depend on chirality.

In these expressions, $q(s)$ is the Laplace transform (when it exists) of any function $q(t)U(t)$. For instance, for a harmonic pulse $\exp(\mathrm{i}\omega t)U(t)$ we have $q(s) = (s + \mathrm{i}\omega)^{-1}$ while for a dipole radiating a sequence of j square pulses [13], each with a duration T

$$q(s) = (q - \mathrm{e}^{-2jsT})/s(1 + \mathrm{e}^{sT}). \tag{22}$$

Substituting (22) into (20) or (21) gives the electromagnetic field radiated by a dipole feeded with digital pulses.

4.2. Guided waves

We consider the propagation in the direction $z \geqslant 0$ of an electromagnetic wave launched at $t = 0$ in a medium with the constitutive relations (3) and bounded by perfectly conducting planes located at $x = 0$ and $x = a$. The only non-null component Π_y of the Hertz potential vector satisfies the 2D wave equation $(\partial_x^2 + \partial_z^2 - p^2)\Pi_y = 0$ with the solutions null for $x = 0, a$

$$\Pi_y(x, z, s) = A \sin k_1 x \exp(-k_3 z) \tag{23}$$

$$k_1 = l\pi/a \qquad k_3 = (p^2 + k_1^2)^{1/2} \tag{24}$$

l is an arbitrary integer and $A = A_1 + \mathrm{i}A_2$ a complex constant. Substituting (23) into (16) gives

$$\begin{aligned} \sqrt{\varepsilon}E_{x,\pm} &= \pm A_2 k_3 p \sin k_1 x \exp(-k_3 z) \\ \sqrt{\varepsilon}E_{y,\pm} &= A_1 p^2 \sin k_1 x \exp(-k_3 z) \\ \sqrt{\varepsilon}E_z \pm &= \pm A_2 k_1 p \cos k_1 x \exp(-k_3 z). \end{aligned} \tag{25}$$

The components of $\mathbf{H}$ are obtained as previously by changing $A_1, \pm A_2$ into $A_2, \mp A_1$. So, in a chiral wave guide, the TE and TM waves are coupled. We remind that in (25) p and k_3 are function of s.

4.3. Diffraction of a harmonic pulse

We consider a harmonic pulse $\exp[\mathrm{i}\omega t + \mathrm{i}kx\cos\theta_0 + \mathrm{i}kz\cos\theta_0]U(t)$ impinging with the angle of incidence θ_0 on a perfectly conducting half plane $z = 0$, $x > 0$, immersed in a medium with a constant refractive index and chirality parameter. Generalizing somewhat the Bateman presentation [5] of the Sommerfeld diffraction theory, we write the non-null component Π_y of the Hertz potential

$$\Pi_y = AU_1 + BU_2 \tag{26}$$

in which using the polar coordinates $x = r\cos\theta$ and $z = r\sin\theta$, the functions U_1 and U_2, are

$$U_1 = (s + \mathrm{i}\omega)^{-1}\exp[\mathrm{i}knr\cos(\theta - \theta_0)]\,\mathrm{e}^{\mathrm{i}\pi/4}\int_{-\infty}^{T}\exp(-\mathrm{i}u^2)\,\mathrm{d}u$$

$$U_2 = (s + \mathrm{i}\omega)^{-1}\exp[\mathrm{i}knr\cos(\theta + \theta_0)]\,\mathrm{e}^{\mathrm{i}\pi/4}\int_{-\infty}^{T}\exp(-\mathrm{i}u^2)\,\mathrm{d}u \tag{27a}$$

$$T_\pm = (2nkr)^{1/2}\cos[(\theta \pm \theta_0)/2] \tag{27b}$$

while the constants A, B are determined by the boundary conditions on the perfectly conducting screen

$$E_{y|S} = 0 \tag{28a}$$

$$H_{y|S} = 0 \tag{28b}$$

accordingly as the electric or magnetic vector is parallel to the edge of the screen.

Now from (16) we get

$$\begin{aligned}\sqrt{\varepsilon}E_{x,\pm} &= \pm p\,\mathrm{Im}\,(\partial_z\Pi_y)\\ \sqrt{\varepsilon}E_{y,\pm} &= -p^2\,\mathrm{Re}\,(\Pi_y)\\ \sqrt{\varepsilon}E_{z,\pm} &= -_{+} p\,\mathrm{Im}\,(\partial_x\Pi_y).\end{aligned} \tag{29}$$

For $\sqrt{\mu}\mathbf{H}$ change p and Re into $-p$ and Im. Substituting (26) into (29) and using (27a) one checks easily that condition (28a) is satisfied for $B = -A$. For (28b) we first remark that according to the Maxwell equations (12)

$$\partial_y(H_z + \xi E_z) = \partial_z(H_y + \xi E_y) = pE_x \tag{30}$$

however, since the electromagnetic field does not depend on y, condition (28b) is equivalent to

$$(pE_x + \xi\partial_z E_y)_{|s} = 0 \tag{31a}$$

that is using (29)

$$(\mathrm{Im}\, - \xi\,\mathrm{Re}\,)(\partial_z\Pi_y)|_s = 0 \tag{31b}$$

$\partial_z\Pi_y$ reduces to $\partial_\theta\Pi_y$ for $\theta = 0,\ 2\pi$. Then, using (27a) and (27b) a simple calculation shows that condition (28b) is satisfied for $A = B$.

The Bateman technique for solving the Sommerfeld problem in a bi-isotropic medium is discussed in [14] for an incident harmonic plane wave.

Remark. These applications show that the presence of chirality does not affect very much the mathematical development of Hertz potentials, and this is essentially because we have considered reciprocal chiral media (the existence of more general media is still controversial) in which the two components of the electromagnetic field have the same amplitude. However, from a physical point of view, the importance of chirality should not be minimized: for instance, TE and TM waves become coupled in a chiral wave guide and the role of chiral material in the diffraction of radar waves is important.

In some of these applications where ε, μ, ξ are assumed constant, the case where they depend on s is easy to tackle, only the calculation of the inverse Laplace transform is more difficult.

5. Inverse Laplace transform

The examples of section 4 prove that the Laplace transform is a powerful tool in solving initial value problems in the s-domain. However, except in some cases where changing s into $\mathrm{i}\omega$ provides a clear picture of the solution in the frequency domain, one is generally more interested by the time behaviour of the solution. So, we need to discuss how to perform the inverse Laplace transform, leaving aside the very particular case where it can be found in a table [15].

We deal with a scalar field $F(x, s)$, noting that the solution of the Laplace transformed wave equation has the general form (see section 4)

$$F(\boldsymbol{r}, s) = f(\boldsymbol{r}, s)\,\mathrm{e}^{-g(r,s)} \tag{32}$$

then, the inverse Laplace transform of (32) is supplied by the Bromwich integral [16]

$$F(\boldsymbol{r}, t) = (2\pi\mathrm{i})^{-1}\int_{B_r} \mathrm{e}^{st-g(r,s)} f(\boldsymbol{r}, s)\,\mathrm{d}s. \tag{33}$$

The contour B_r is a straight line from $L - \mathrm{i}^\infty$ to $L + \mathrm{i}^\infty$ where $L \geqslant 0$ is real and all the singularities of the integrand are on the left of L. Generally the singularities of $st - g(r, s)$ are branch points while those of $f(r, s)$ may be poles and branch points. The inverse Laplace transform is an ill-posed problem in Hadamard's sense (see [17] for a recent discussion of this point) so that one has to be careful. We list three classes of techniques to tackle (33).

(1) When only singularities of $f(r, s)$ are poles, one may look for an asymptotic approximation of (33) with the help of saddle-point methods as illustrated for instance by Oughstun *et al* [18] in their analysis of the Sommerfeld–Brillouin problem. A recent review of these asymptotic techniques is given in [19]. The situation is more difficult when $f(r, s)$ has branch points, one has in this case to deform carefully the Bromwich contour to get approximate equivalent contours [20].

(2) There exist now powerful and efficient numerical codes to perform inversion efficiently. They use either the fast Fourier transform [21] or more specific algorithms [22].

(3) For real s, the inverse Laplace transform may be obtained by some kind of regularization technique [23] or by an approximation of the Widder inverse formula [24]

$$F(t) = \lim_{n\Rightarrow\infty}[s^{n+1}/n!(-\mathrm{d}/\mathrm{d}s)^n f(s)]_{s=n/t}. \tag{34}$$

This formula gives numerical relations between the image and the original at any point where $F(t)$ has limited total fluctuation. It may be expected that in considering (34) for finite n, one obtains a set of approximations for $F(t)$. We discuss these approximations later.

6. Conclusion

Maxwell's equations have been the basis of electromagnetic theory for a century. They were successful in providing solutions with sinusoidal time variation which were especially convenient since technology could mainly generate sequences of harmonic waves. However, the recent blossoming of digital technology makes it necessary to look for solutions of Maxwell's equations which depend arbitrarily on time. As shown here, the Hertz–Debye potential technique coupled with the Laplace transform is an elegant answer to this requirement. Although our investigations were limited to initial value problems one could tackle similarly initial-boundary value problems and in this case it may be interesting to work with a covariant Laplace transform [25], especially when electromagnetic pulses are propagated in a moving media.

As shown here, the complex formalism of electromagnetism [4] makes it possible to unify the analysis of wave propagation is isotropic and bi-isotropic media. However, the constitutive relations to be used in a medium sustaining arbitrary and in particular digital pulses are still a challenge since the time behaviour of materials is rarely known under transient conditions. It is unfortunate that few studies have been carried out in this domain.

Appendix

We obtain from (14)

$$\mathrm{curl}\,\boldsymbol{U} = -ns/c\,\mathrm{curl}\,\boldsymbol{\Pi} + \mathrm{i}[\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\Pi} - \Delta\boldsymbol{\Pi}] \tag{A1}$$

$$\mathrm{grad}\,\boldsymbol{V} = -\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\Pi} + ns/c\,\mathrm{grad}\,L \tag{A2}$$

so, taking into account (13)

$$-ns/c\boldsymbol{U} + \mathrm{grad}\,V = n^2s^2/c^2\boldsymbol{\Pi} - \mathrm{i}ns/c\,\mathrm{curl}\,\boldsymbol{\Pi} - \mathrm{grad}\,\mathrm{div}\,\boldsymbol{\Pi}. \tag{A3}$$

The comparison of (A1) multiplied by i with (A3) gives the result.

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